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CESIFO WORKING PAPER NO. 3653

CATEGORY 7: MONETARY POLICY AND INTERNATIONAL FINANCE

NOVEMBER 2011

CESifo *Working Paper No. 3653*

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Abstract

This paper analyses two well-known features of interest rates, namely their time dependence and their cyclical structure. Specifically, it focuses on the monthly Euribor rate, using monthly data from January 1994 to May 2011. Models based on fractional integration at the long run or zero frequency, although adequately describing the persistent behaviour of the series, do not take into account its cyclical structure. Therefore, a more general cyclical fractional model is considered. Future directions for research in this context are also discussed.

JEL-Code: C220, E300.

Keywords: Euribor rate, time dependence, cyclical behaviour.

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October 2011
The first-named author gratefully acknowledges financial support from the the Ministerio de Ciencia y Tecnología (ECO2008-03035 ECON Y FINANZAS, Spain) and from a PIUNA Project from the University of Navarra.

1. Introduction

The choice of appropriate models for interest rates is a hotly debated issue, as it is unclear whether they should be modelled as I(0) or I(1) processes. A well-known stylised fact is their high persistence. This could be approximated by an AR(1) (I(0)) process with a root close to 1. Alternatively, unit root (I(1)) processes could be considered. Earlier studies typically focused on whether interest rates can be characterised as an I(0) or I(1) series. For instance, Cox, Ingersoll and Ross (1985) concluded that the short-term nominal interest rate is a stationary and mean-reverting I(0) process, whereas authors such as Campbell and Shiller (1987) assumed a unit root. The drawback of the I(0) models is that they imply long-rates which are not volatile enough (Shiller, 1979) whereas the problem with the I(1) models is that they imply that the term premium necessarily increases with bond maturities (Campbell, Law and MacKinlay, 1997).[1]

More general I(d)-type specifications provide additional flexibility to model such persistent behaviour. In the last two decades some studies have taken this approach. For instance, Shea (1991) investigated the consequences of long memory in interest rates for tests of the expectations hypothesis of the term structure. He found that allowing for the possibility of long memory significantly improves the performance of the model, even though the expectations hypothesis cannot be fully resurrected. In a related paper, Backus and Zin (1993) observed that the volatility of bond yields does not decline exponentially when the maturity of the bond increases; in fact, the decline is hyperbolic, consistently with a fractionally integrated specification. Lai (1997) and Phillips (1998) provided evidence based on semiparametric methods that ex-ante and ex-post US real interest rates are fractionally integrated. Tsay (2000) employed a fractionally ARIMA (ARFIMA) model and provided evidence that the US real interest rate can be described as an I(d)

[1] Recently, Gil-Alana and Moreno (2008) have proposed a fractional integration model for the short-term interest rate and the term premium.

process. Further evidence can be found in Barkoulas and Baum (1997), Meade and Maier (2003) and Gil-Alana (2004a,b). Couchman, Gounder and Su (2006) estimated ARFIMA models for ex-post and ex-ante interest rates in sixteen countries. Their results suggest that for the majority of these countries the fractional differencing parameter lies between 0 and 1 and is considerably smaller for the ex-post real rates compared with the ex-ante ones.

Another well-known feature of interest rates is their cyclical structure (see, e.g., Kessel, 1965) that is not well captured by I(0), I(1) or I(d) models, the last two of which are all characterised by a spectral density function which is unbounded at the origin (i.e. the zero frequency). Typically, interest rates exhibit a peak in the periodogram at non-zero (as opposed to zero) frequencies indicating a certain degree of cyclical behaviour. This cyclical structure can be captured by a simple AR(2) process with complex roots; however, such a process is characterised by a very rapid decay in the autocorrelations, which is not consistent with the high level of persistence observed in interest rates. A long memory cyclical I(d) model can instead overcome this limitation, and the aim of the present study is to propose and evaluate such a model for the Euribor rate.

The paper is organised as follows. Section 2 outlines the concept of long-range dependence. Section 3 discusses the main features of the data. Section 4 presents the estimation and testing results. Section 5 summarises the main findings and suggests some extensions.

2. Long-range dependence and cycles

The analysis in this paper is based on the concept of long-range dependence. Given a covariance stationary process $\{x_t, t = 0, \pm 1, \dots \}$, with autocovariance function $E(x_t - Ex_t)(x_{t-j} - Ex_t) = \gamma_j$, according to McLeod and Hipel (1978), x_t displays the property of long memory if

$$\lim_{T\to\infty} \sum_{j=-T}^{T} \left|\gamma_j\right|$$

is infinite. An alternative definition, based on the frequency domain, is the following. Suppose that x_t has an absolutely continuous spectral distribution function, and hence a spectral density function, denoted by $f(\lambda)$, and defined in terms of the autocovariances as

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda\, j, \qquad -\pi < \lambda \le \pi.$$

Then x_t displays the property of long memory if the spectral density function has a pole at some frequency λ in the interval $[0, \pi]$, i.e.,

$$f(\lambda) \;\to\; \infty, \qquad as\ \lambda \to \lambda^*, \qquad \lambda^* \in\ [0, \pi).$$

Most of the empirical literature has focused on the case where the singularity or pole in the spectrum occurs at the smallest (zero) frequency. This is the standard case of I(d) models of the form:

$$(1 - L)^d x_t = u_t, \qquad t = 0, \pm 1, \ldots, \tag{1}$$

where L is the lag operator ($Lx_t = x_{t-1}$) and u_t is I(0) defined, for the purpose of the present paper, as a covariance stationary process with a spectral density function that is positive and bounded at all frequencies. Thus, if d = 0 in (1), $x_t = u_t$, the process is short memory and it could be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially; however, it could decay at a much slower rate than exponentially (in fact, hyperbolically) if d is positive. When d = 0 x_t is also said to be "*weakly autocorrelated*" as opposed to the case of "*strongly autocorrelated*" if d > 0. Moreover, if $0 < d < 0.5$, x_t is still covariance stationary, but its lag-j autocovariance γ_j decreases very slowly, at the rate of j^{2d-1} as $j \to \infty$, and so the γ_j are absolutely non-summable.[2] The

[2] Note that these two conditions, which can be expressed as $\gamma_j \sim c\, j^{2d-1}$ as $j \to \infty$, and $f(\lambda) \sim c^*\lambda^{-2d}$ as $\lambda \to 0^+$, for $0 < c, c^* < \infty$, are not always equivalent but Zygmund (1995, Cap. V, Section 2) and Yong (1974) in a more general case give conditions under which both expressions are equivalent.

variable x_t is then said to have long memory given that $f(\lambda)$ is unbounded at the origin.[3] Also, as d in (1) increases beyond 0.5 and through 1 (the unit root case), x_t can be viewed as becoming "*more nonstationary*" in the sense, for example, that the variance of the partial sums increases in magnitude. Processes of the form given by (1) with positive non-integer d are called fractionally integrated, and when u_t is ARMA(p, q) x_t is known as a fractionally ARIMA (or ARFIMA) model. This type of model provides a higher degree of flexibility in modelling low frequency dynamics which is not achieved by non-fractional ARIMA models.

As previously mentioned the above processes are characterised by a spectral density function which is unbounded at the zero frequency. However, a process may also display a pole or singularity in the spectrum at a frequency away from zero. In this case, it may still display the property of long memory but the autocorrelations exhibit a cyclical structure that is decaying very slowly. This is the case of the Gegenbauer processes defined as:

$$(1 - 2\cos w_r L + L^2)^d x_t \; = \; u_t, \qquad t \; = \; 1, 2, \ldots, \tag{2}$$

where w_r and d are real values, and u_t is I(0). For practical purposes we define $w_r = 2\pi r/T$, with r = T/s, and thus s will indicate the number of time periods per cycle, while r refers to the frequency that has a pole or singularity in the spectrum of x_t. Note that if r = 0 (or s = 1), the fractional polynomial in (2) becomes $(1 - L)^{2d}$, which is the polynomial associated with the common case of fractional integration at the long-run or zero frequency. This type of process was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Giraitis and Leipus (1995), Chung (1996a,b) and Dalla and Hidalgo (2005) among many others.

[3] Such processes were first considered in the 1960s by Granger (1966) and Adelman (1965) who pointed out that for most aggregate economic time series the spectral density function increases sharply as the frequency approaches zero and that differencing the data leads to overdifferencing at the zero frequency.

Gray et al. (1989, 1994) showed that the polynomial in (2) can be expressed in terms of the Gegenbauer polynomial, such that, denoting $\mu = \cos w_r$, for all $d \neq 0$,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j , \qquad (3)$$

where $C_{j,d}(\mu)$ are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d}(\mu) = 1, \qquad C_{1,d}(\mu) = 2\mu d,$$

$$C_{j,d}(\mu) = 2\mu\left(\frac{d-1}{j}+1\right)C_{j-1,d}(\mu) - \left(2\frac{d-1}{j}+1\right)C_{j-2,d}(\mu), \quad j = 2, 3, ...,$$

(see, for instance, Magnus et al., 1966, Rainville, 1960, etc. for further details on Gegenbauer polynomials). Gray et al. (1989) showed that x_t in (2) is (covariance) stationary if $d < 0.5$ for $|\mu = \cos w_r| < 1$ and if $d < 0.25$ for $|\mu| = 1$.[4]

These processes (of the form given by the equations (1) and (2)) will be employed for the empirical analysis in Section 4.

3. The Euribor Rate: Empirical Features

The series analysed is the monthly Euribor rate for the time period 1994m1 - 2011m5. Figure 1 suggests that it might have a cyclical structure, although this is less apparent in the case of the first differenced data.

[Figures 1 – 3 about here]

Figure 2 displays the first 100 values of the sample correlogram for the original and the first differenced data. A cyclical structure again seems to be present, although a longer time appears to be required to complete a cycle, at least for the original data. The periodogram of the series in both levels and first differences is displayed in Figure 3. One can see in both cases a peak at frequency 3 which corresponds to T/3, i.e. approximately

[4] Note that if $|\mu| < 1$ and d in (2) increases beyond 0.5, the process becomes "more nonstationary" in the sense, for example, that the variance of the partial sums increases in magnitude.

70 periods per cycles, which is 5.83 years per cycle. This may be related to the existence of cycles in economic activity, business cycles generally being defined as having a periodicity ranging between 5 and 8 years.[5]

4. Empirical Results

As a first step we investigate the degree of persistence of the series by estimating the order of integration. Initially, we carry out standard unit root tests (ADF, Dickey and Fuller, 1979; PP, Phillips and Perron, 1988; Elliot et al., 1996; and others). The results (not reported here) strongly support the hypothesis that the series is I(1), although this finding may simply reflect the low power of these tests against fractional alternatives.[6]

We consider the case of fractional integration at the long run or zero frequency, and specifically the following model,

$$y_t = \alpha + \beta t + x_t; \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, \qquad (4)$$

where y_t is the observed time series, α and β are unknown coefficients corresponding to the intercept and a linear time trend, d is a real value number and u_t is assumed to be I(0) as previously defined. We estimate d using a Whittle parametric function in the frequency domain, allowing u_t to follow different processes. First, we assume that it is white noise. Then, we model it as a weakly autocorrelated process, in particular AR(1), AR(2), seasonal (monthly) AR(1), and finally we use the exponential spectral model of Bloomfield (1973). The latter is a non-parametric approach that produces autocorrelations decaying exponentially as in the AR(MA) case. In addition to the estimation, we carry out Lagrange Multiplier (LM) tests of the null hypothesis:

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[5] Burn and Mitchell (1946), Romer (1986, 1994), Stock and Watson (1998), Diebold and Rudebusch (1992), Canova (1998), Baxter and King (1999), King and Rebelo (1999) among others showed that the average length of the cycle is approximately six years.

[6] Diebold and Rudebusch (1991), Hassler and Wolters (1994), and Lee and Schmidt (1996) inter alia have shown that standard unit root tests have very low power against fractional alternatives.

$$H_o : d = d_o, \quad (5)$$

in (4) for a grid of real-values d_o for each type of I(0) disturbances using a methodology devised by Robinson (1994).[7]

[Insert Table 1 about here]

Table 1 displays the estimates of d based on the Whittle function along with the 95% confidence band of the non-rejection values of d using Robinson's (1994) parrametric approach. We present the results for the three standard cases of no regressors (i.e., $\alpha = \beta = 0$ in (4), an intercept (α unknown and $\beta = 0$), and an intercept with a linear time trend in (4). We note first that all the estimated values of d are above 1 and this happens for the three types of regressors used and the alternative ways of modelling the I(0) disturbances. In fact, there is only one case where the unit root null cannot be rejected, namely that with no regressors and Bloomfield-type disturbances. In all the other cases, the unit root null hypothesis is rejected in favour of higher degrees of integration, implying a high degree of dependence in the data when using the specification given by equation (4).

Note, however, that the above specification does not consider the possibility of cycles. In fact, only the AR(2) model for the disturbances might incorporate cycles if the roots of the short-run (AR) dynamics are of a complex form. Alternatively, another specification can be considered, based on the Gegenbauer processes as described in Section 2.

[Insert Table 2 about here]

Table 2 displays the estimates of d based on the following model,

$$y_t = \alpha + \beta t + x_t; \quad (1 - 2\cos w_r L + L^2)^d x_t = u_t, \quad t = 1, 2, \quad (6)$$

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[7] This method is based on the Lagrange Multiplier (LM) principle. A Wald testing approach (Lobato and Velasco, 2007) was also implemented, using the Whittle estimates of d. The results were completely in line with those reported here.

with $w_r = 2\pi T/r$, r = 2, ..., T/2, again for the three cases of no regressors, an intercept, and an intercept with a linear trend, and using again the Whittle function along with the parametric LM method of Robinson (1994). Several features are noteworthy. First, the estimated value of r (not reported) is 70, which is consistent with the peak in the periodogram detected in Figure 3. Also, all the estimated values of d are now strictly smaller than 1 implying mean reversion with respect to this cyclical frequency. The results are in virtually all cases in the interval [0.5, 1) suggesting that the series is nonstationary, the only cases with values of d smaller than 0.5 being obtained for the approximation with the Bloomfield disturbances.

[Insert Table 3 about here]

In Table 3 we report the coefficient estimates of the selected models according to the selected specification for the deterministic terms and the disturbances. The time trend is required in all cases with a significant negative trend of about -0.013. The estimated value of d is about 0.67 for white noise, AR(2) and seasonal AR(1) disturbances. It is slightly smaller (0.65) for in the AR(1) case and about 0.54 with the exponential spectral model of Bloomfield (1973). We also see that the short-run parameters are very close to 0 in all cases, suggesting that no additional (short-run) time dependence is required when modelling this series.

[Insert Figures 4 and 5 about here]

The upper and lower panels of Figure 4 display the deterministic trend and the detrended series respectively; the cyclical structure is apparent, especially if we look at the correlogram and the periodogram, which are both displayed in Figure 5. A peak at frequency 3 is still very noticeable in the case of the periodogram.

[Insert Figure 6 about here]

The correlogram and periodogram of the estimated residuals, presented in Figure 6, suggest that the cyclically I(d) model with a linear trend may be an adequate specification for this series. In fact, serial correlation tests provide strong evidence of no correlation in the estimated residuals.

Finally, in order to establish whether business cycles can account for the cyclical behaviour of the Euribor, we include in the model a monthly industrial production (IP) index for the euro area as a whole (data source: Eurostat), and estimate the following specification:

$$y_t = \alpha + \beta \, gr_{t-k} + x_t \, ; \qquad (1 - 2\cos w_r L + L^2)^d \, x_t = u_t \, , \qquad t = 1,2,$$

where gr_{t-k} is the growth rate of IP (computed as the first differences of the log-transformed data) with k = 0, 1, 2, 3, 4, 5 and 6. The procedure outlined above (see Robinson, 1994) can still be implemented under the assumption that gr_{t-k} is a (weakly) exogenous variable. The estimation results for d, for different types of I(0) disturbances u_t, were very similar to those reported in Table 2, with the values ranging in the interval (0.5, 1) in all cases. However, the β coefficient was not found to be statistically significant for any value of k, implying that the growth rate is not a relevant variable to explain the behaviour of the Euribor. When examining the correlogram and the periodogram of the growth rate series, we notice that the peak in the periodogram now occurs at the 5th frequency, implying cycles of about T/5 = 41.4 = 3.45 years/cycle, which are considerably shorter than in case of the Euribor, thus ruling out the possibility of fractional cointegration at a given frequency. Further analysis should be conducted to find a variable exhibiting a pole at the same frequency in the spectrum as the Euribor. In such a case, the possibility of fractional cointegration at a given cyclical frequency can be examined. This is an issue that has not been extensively investigated (Gil-Alana, 2009 is one of the few exceptions). The idea is that two series which are cyclically I(d) as in (2) with the peak in the spectrum occurring at the same frequency (say r) and the same degree of integration (d) may be cointegrated in the sense that there exists a linear combination of the two variables which is (cyclically) fractionally integrated of a smaller order than the original series at the same frequency r. Alternatively, weakly exogenous regressors that might influence the Euribor can also be included in a regression model where the errors are cyclically fractionally integrated.

5. Conclusions

This paper analyses two well-known features of interest rates, namely their time dependence and their cyclical structure. Specifically, it focuses on the Euribor rate, using monthly data from January 1994 to May 2011. Models based on fractional integration at the long-run or zero frequency, although adequately describing the persistent behaviour of the series, do not take into account the cyclical structure of the series. Therefore, a more general cyclical fractional model is considered. We use an approach that is based on the Gegenbauer processes and that produces autocorrelations decaying hyperbolically with a cyclical pattern. The results indicate that this model fits the data well, with an order of integration ranging between 0.5 and 1, which indicates nonstationary mean-reverting behaviour. The model implies that the cycle repeats itself every 6 years approximately, which might be related with the business cycles underlying the economy. Future research will focus on finding economic variables that might explain the cyclical structure of the Euribor and be possibly cyclically fractionally cointegrated with this variable.

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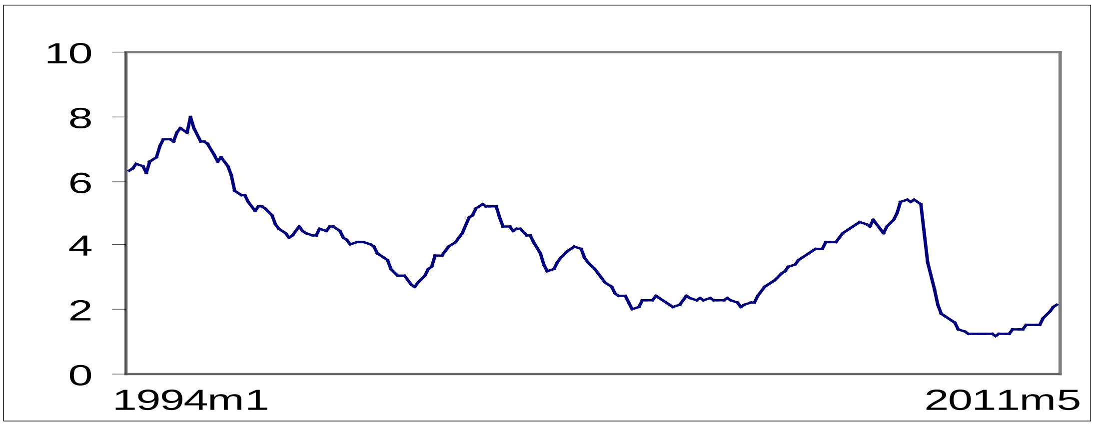
Figure 1: Time series data: Euribor and its first differences

i) Original time series data



ii) First differenced data

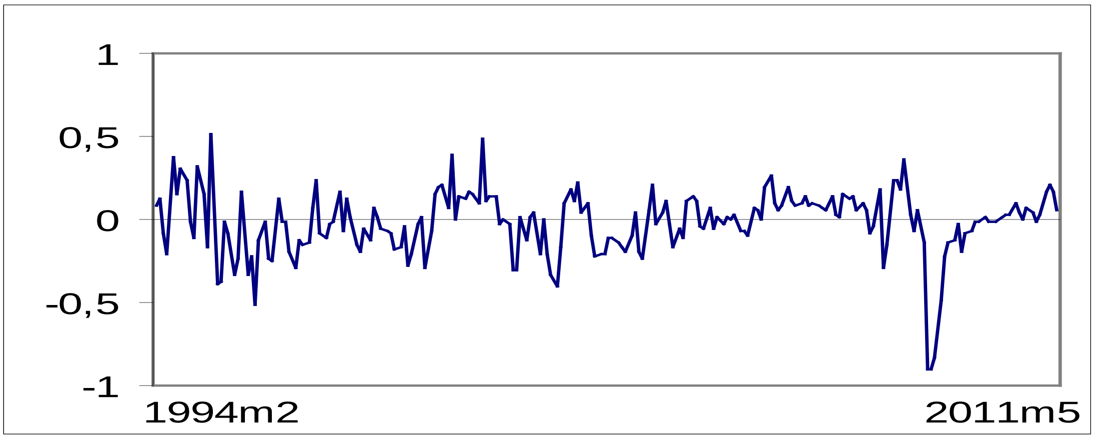
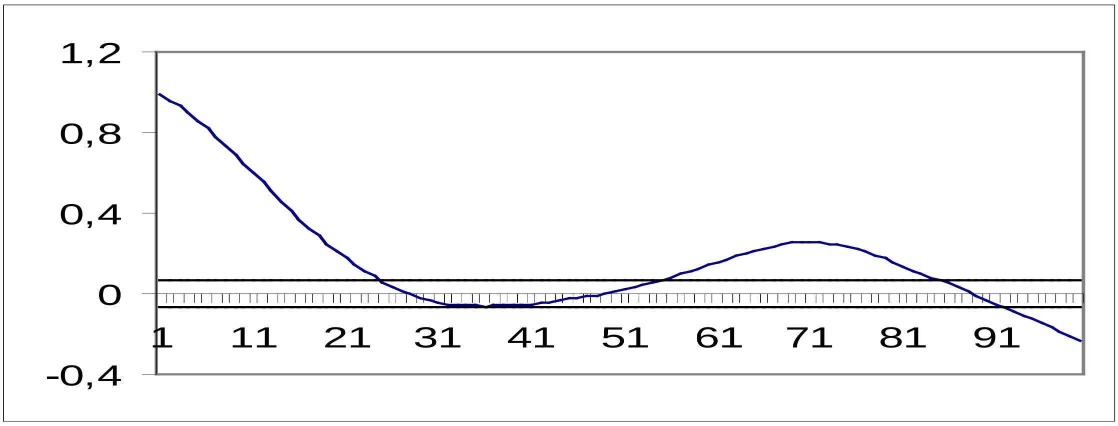
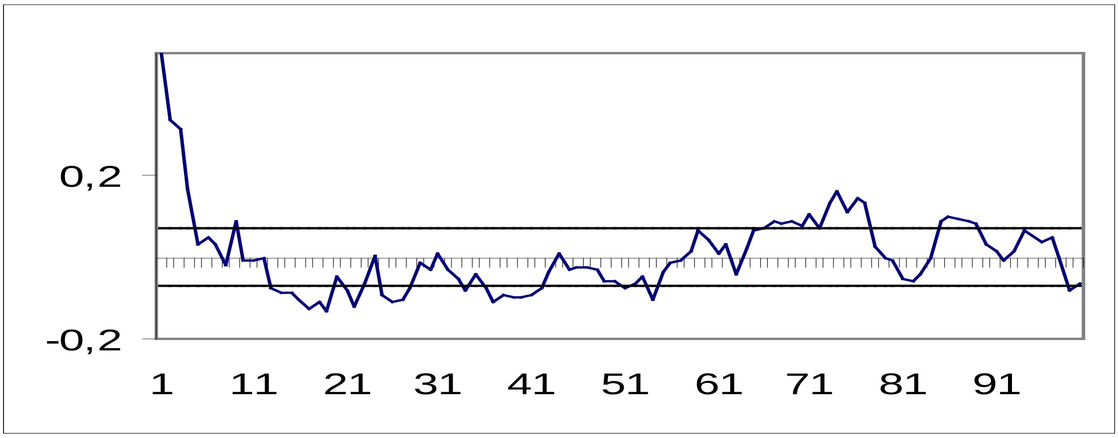


Figure 2: Correlograms of the Euribor in levels and first differences

i) Original time series data



ii) First differenced data



The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.

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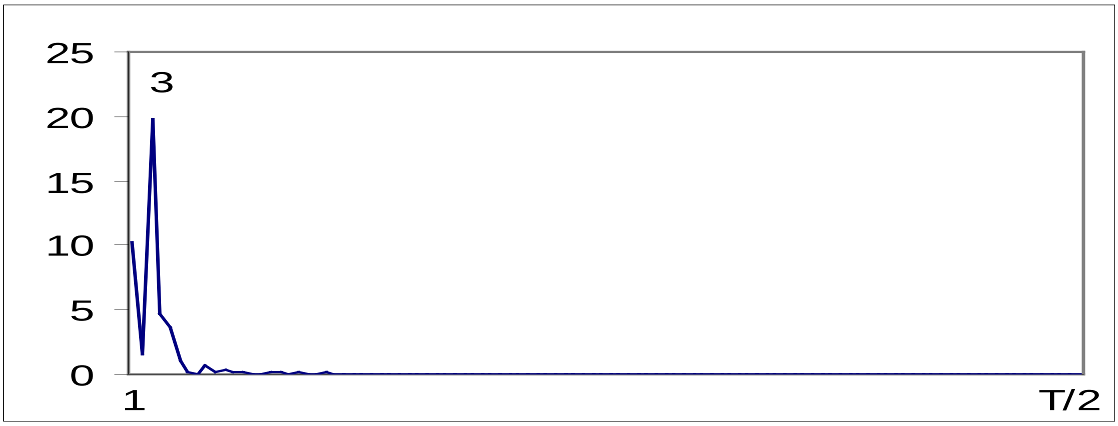
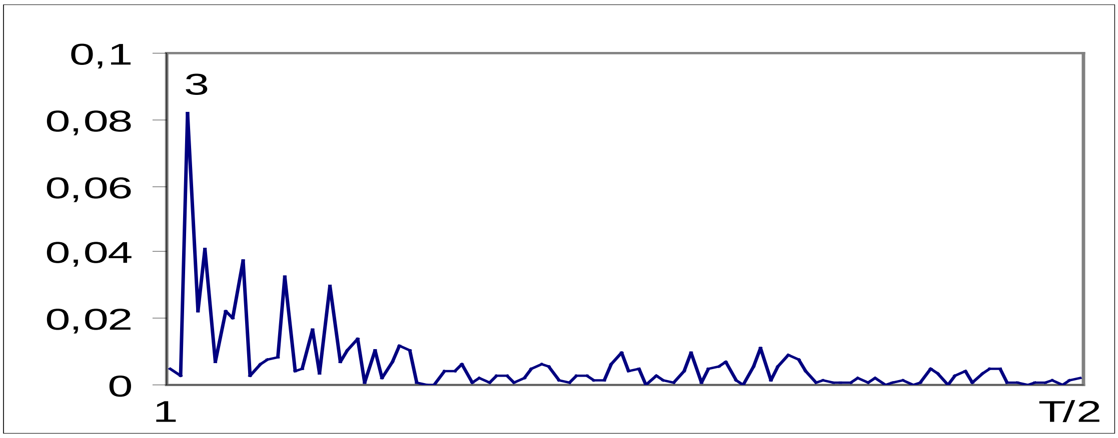
Figure 3: Periodograms of the Euribor and its first differences

i) Original time series



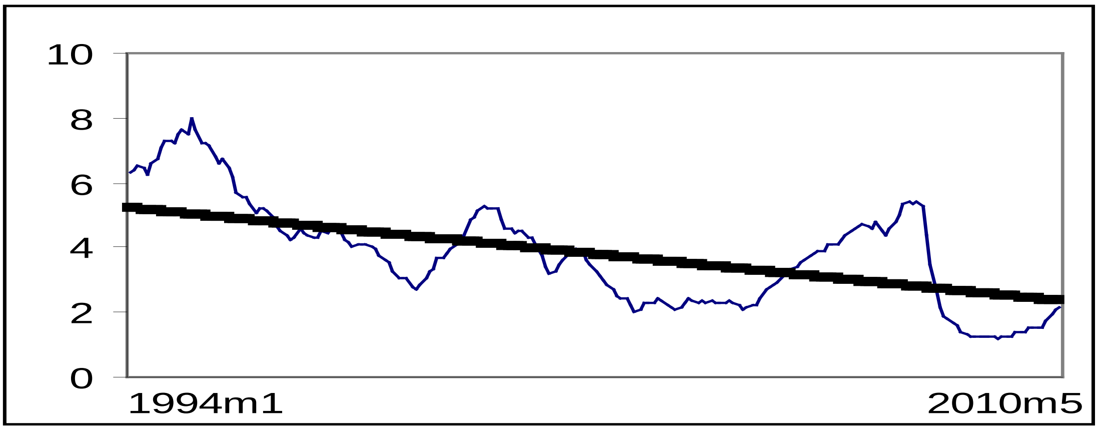
ii) First differenced data



The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, j = 1, …, T/2.

Figure 4: Time series data: Euribor, deterministic trend, and detrended series

i) Original time series data and the time trend



ii) Detrended series

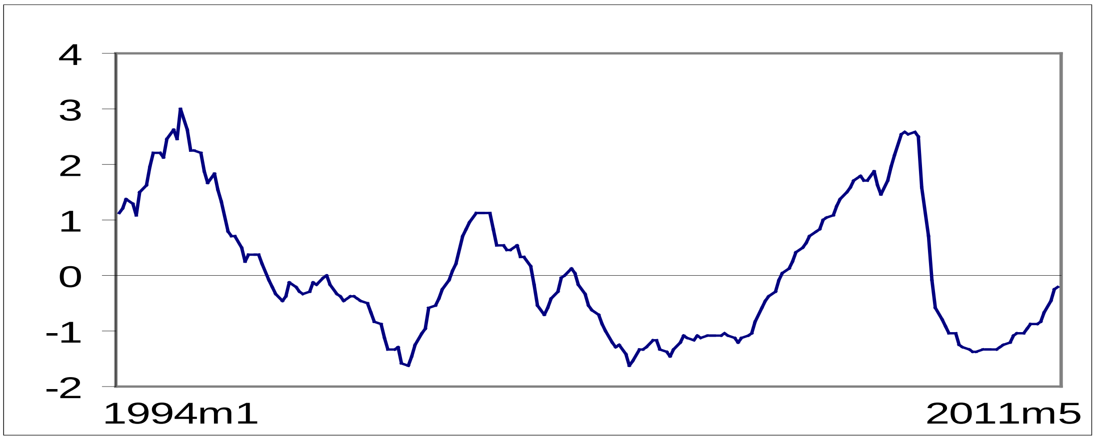
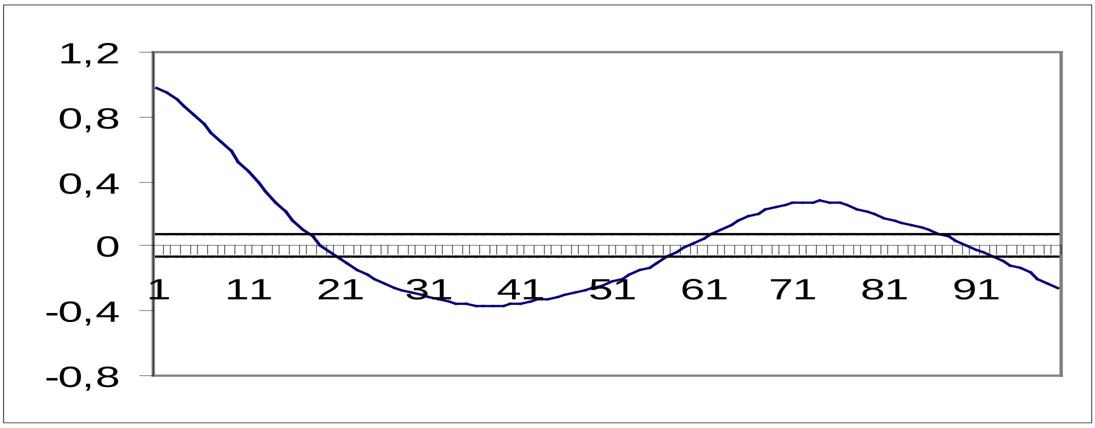
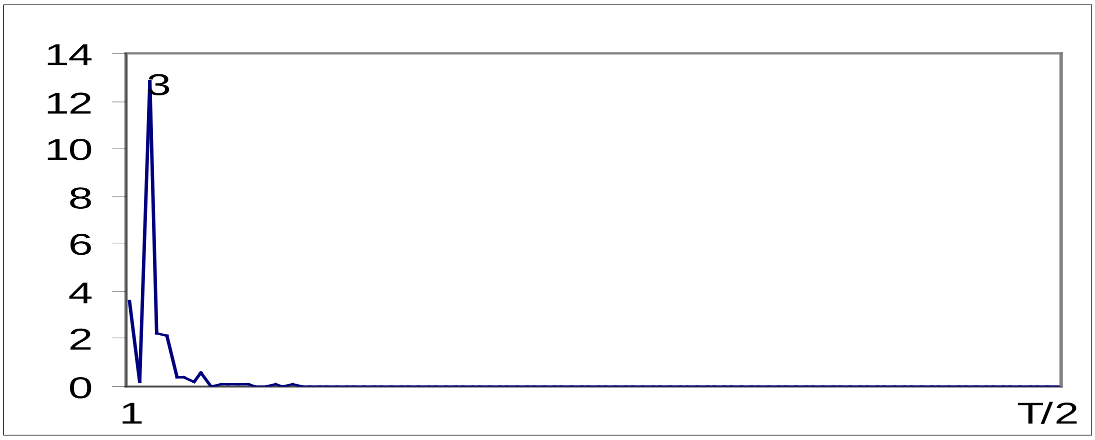


Figure 5: Correlogram and periodogram of the detrended sereis

i) Correlogram*



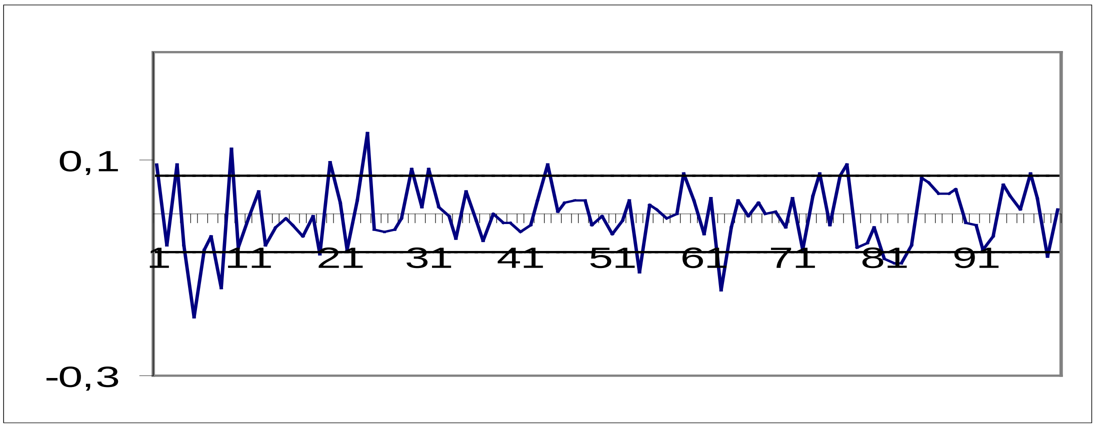
ii) Periodogram**



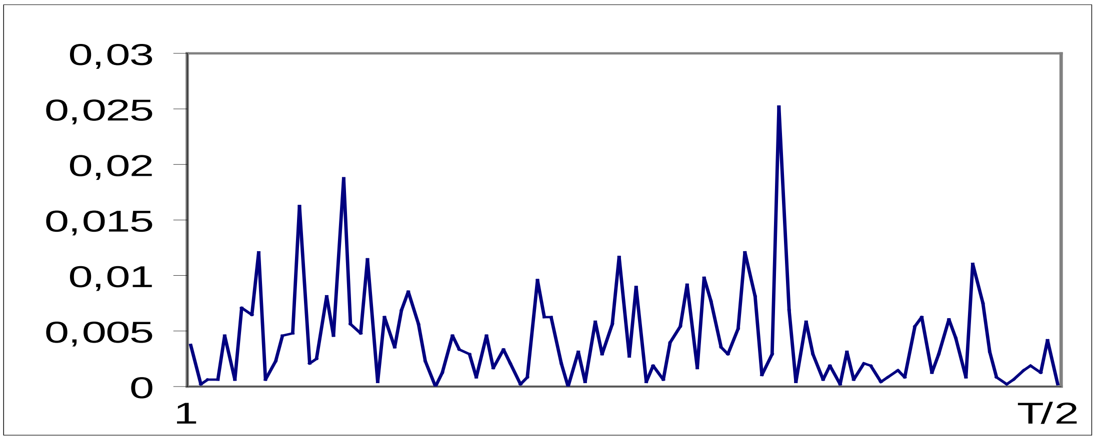
*The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.
**The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, j = 1, …, T/2.

Figure 6: Correlogram and periodogram of the residuals

i) Correlogram*



ii) Periodogram**



*The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.
**The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, \ldots, T/2$.

Table 1: Estimates of d and 95% confidence intervals using model (4)

Frequency	No regressors	An intercept	A linear time trend
White noise	1.105 (1.023, 1.211)	**1.449** **(1.347, 1.578)**	1.448 (1.347, 1.578)
AR (1)	1.428 (1.107, 1.624)	**1.314** **(1.173, 1.533)**	1.316 (1.175, 1.532)
AR (2)	1.973 (1.724, 2.241)	**1.304** **(0.829, 1.939)**	1.304 (0.821, 1.936)
Seasonal AR (1)	1.104 (1.021, 1.210)	**1.453** **(1.349, 1.584)**	1.453 (1.349, 1584)
Bloomfield (m = 1)	1.119 (0.963, 1.333)	**1.283** **(1.069, 1.527)**	1.283 (1.069, 1.527)

In bold the significant coefficients.

Table 2: Estimates of d and 95% confidence interval using model (6)

Disturbances	No regressors	An intercept	A linear trend
White noise	0.762 (0.686, 0.840)	0.695 (0.631, 0.781)	**0.677** **(0.610, 0.766)**
AR (1)	0.737 (0.674, 0.801)	0.671 (0.623, 0.734)	**0.658** **(0.609, 0.722)**
AR (2)	0.760 (0.684, 0.826)	0.683 (0.549, 0.805)	**0.675** **(0.552, 0.797)**
Seasonal AR (1)	0.691 (0.628, 0.777)	0.691 (0.628, 0.776)	**0.678** **(0.612, 0.765)**
Bloomfield (m = 1)	0.484 (0.419, 0.572)	0.474 (0.351, 0.657)	**0.544** **(0.439, 0.707)**

In bold the significant coefficients.

Table 3: Estimates of the coefficients in the selected models using equation (6)

Disturbances	d	α	β	Autoc.1	Autoc.2
White noise	0.677 (0.610, 0.766)	5.22275 (8.644)	-0.01380 (-2.787)	------	-----
AR (1)	0.658 (0.609, 0.722)	5.17654 (9.151)	-0.01348 (-2.920)	0.045	-----
AR (2)	0.675 (0.552, 0.797)	5.21762 (8.733)	-0.1377 (-2.807)	0.036	-0.058
Seasonal AR (1)	0.678 (0.612, 0.765)	5.22533 (8.663)	-0.01382 (-2.788)	0.017	-----
Bloomfield	0.544 (0.439, 0.707)	5.08093 (33.527)	-0.01287 (-10.619)	0.339	-----